Appendices for: Improper Signaling in Two-Path Relay Channels

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Appendix A
Proof of Theorem 2

In this appendix, we provide the proof of [1, Theorem 2]. In fact, this theorem has been proved in [2], however, here we give additionally graphs of the possible configurations of the rate functions $\mathcal{R}_{i,j}\left(p_{\mathrm{r}}^{o}, \mathcal{C}_{x}\right)$ in [1, Eq: (5) and Eq: (8)]. These graphs makes the optimization problem more visually clear for the convenience of the reader.

Proof. For the first case in Fig. 1, we have four different orientations for the minimum pair of rate functions for the two paths. The minimum pair is the two decreasing functions $\mathcal{R}_{i,2}\left(p_{\mathrm{r}}^{o}, \mathcal{C}_{x}\right), \forall i$ and hence, their sum will also be decreasing and the optimal solution is $\mathcal{C}_{x}^{*}=0$. Similar argument applies if the minimum pair is the two increasing functions yielding $\mathcal{C}_{x}^{*}=1$. If the minimum pair is of opposite monotonicity, we need to compute the stationary point of their sum because if there is a maximum on $0<\mathcal{C}_{x}<1$, it must occur at the stationary point calculated from [2, Proposition 3].

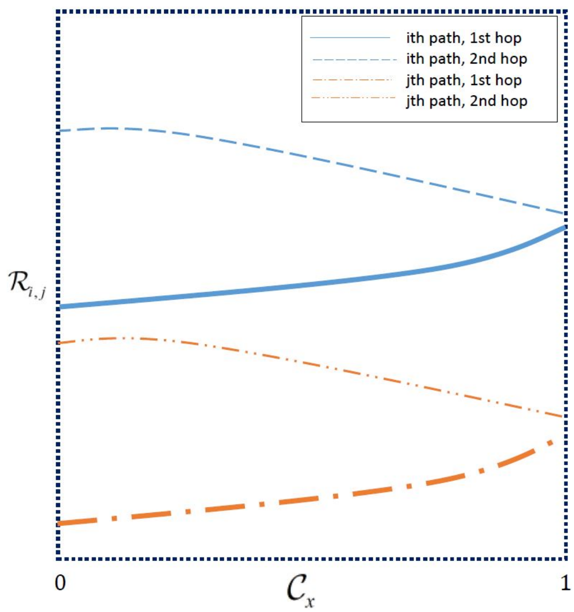
In the second case in Fig. 2, the intersection point, $\mathcal{C}_{i}$, of the two hops rates of the ith path, divides the $\mathcal{C}_{x}$ range into two intervals. In the first interval $0<\mathcal{C}_{x} \leq \mathcal{C}_{i}$, the minimum rate of the ith path is $\mathcal{R}_{i,1}\left(p_{\mathrm{r}}^{o}, \mathcal{C}_{x}\right)$, and in the second interval $\mathcal{C}_{i}<\mathcal{C}_{x} \leq 1$, the minimum rate of the ith path is $\mathcal{R}_{i,2}\left(p_{\mathrm{r}}^{o}, \mathcal{C}_{x}\right)$. For the jth path, we have two different orientations on $0<\mathcal{C}_{x}<1$, either the minimum is the first or the second hop and hence, by a similar argument as in Case 1, the result follows directly.

Finally, in the third case in Fig. 3, we can write the total achievable rate as

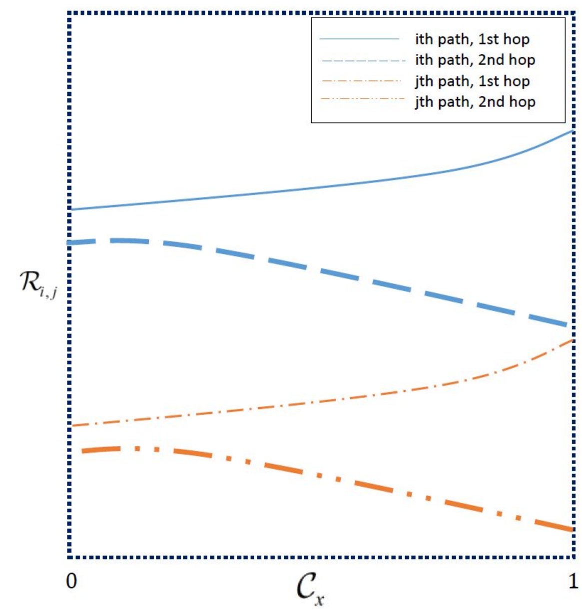
$$\mathcal{R}_{\mathrm{T}}\left(p_{\mathrm{r}}^{o}, \mathcal{C}_{x}\right)=\frac{1}{2} \times \begin{cases} \sum\limits_{i=1}^{2} \mathcal{R}_{i,1}\left(p_{\mathrm{r}}^{o}, \mathcal{C}_{x}\right), & \text{if} \quad 0<\mathcal{C}_{x} \leq \mathcal{C}_{\pi_{1}} \\ R_{\pi_{2},1}\left(p_{\mathrm{r}}^{o}, \mathcal{C}_{x}\right)+R_{\pi_{1},2}\left(p_{\mathrm{r}}^{o}, \mathcal{C}_{x}\right), & \text{if} \quad \mathcal{C}_{\pi_{1}}<\mathcal{C}_{x} \leq \mathcal{C}_{\pi_{2}} \\ \sum\limits_{i=1}^{2} \mathcal{R}_{i,2}\left(p_{\mathrm{r}}^{o}, \mathcal{C}_{x}\right), & \text{if} \quad \mathcal{C}_{\pi_{2}}<\mathcal{C}_{x}<1 \end{cases}. \tag{1}$$

By applying similar arguments as in the previous cases, the result follows directly and this concludes the proof. □

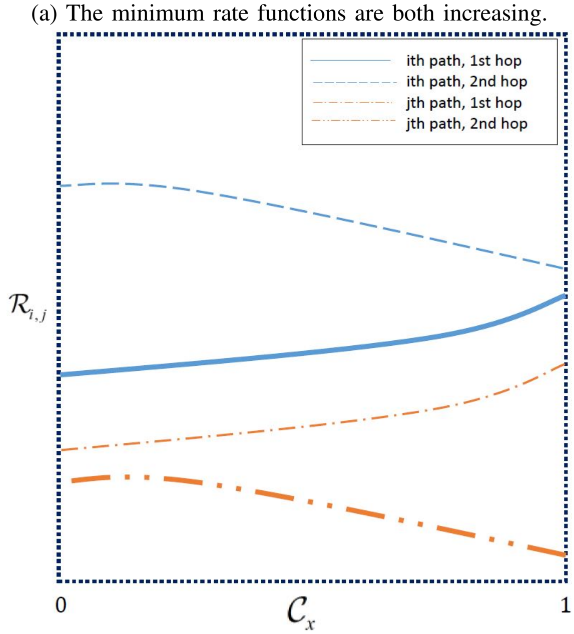
This document contains the appendices for the work in [1] which is submitted to 2017 IEEE International Conference on Communications (ICC) Workshop on Full-Duplex Communications for Future Wireless Networks, Paris, France.



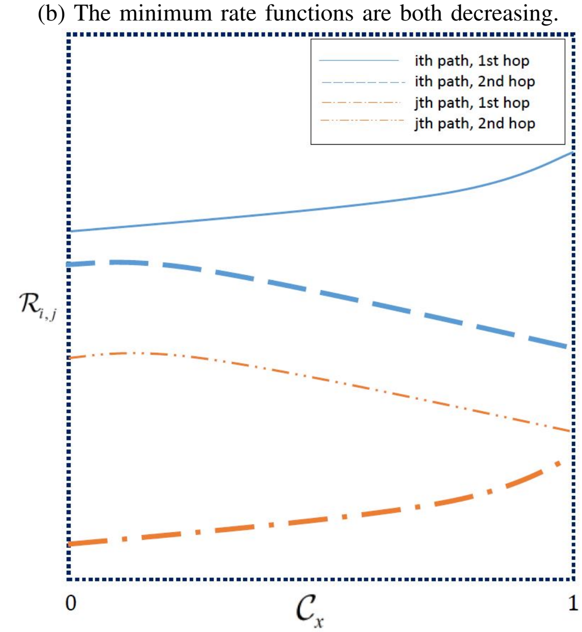
(a) The minimum rate functions are both increasing.



(b) The minimum rate functions are both decreasing.

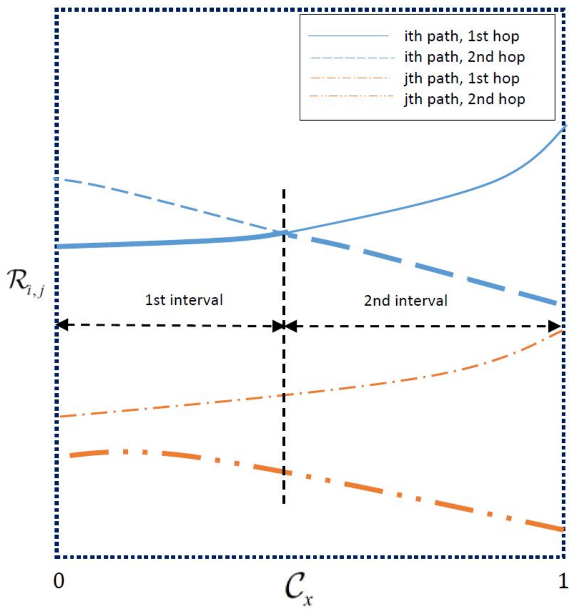


(c) The minimum rate functions are increasing and decreasing.

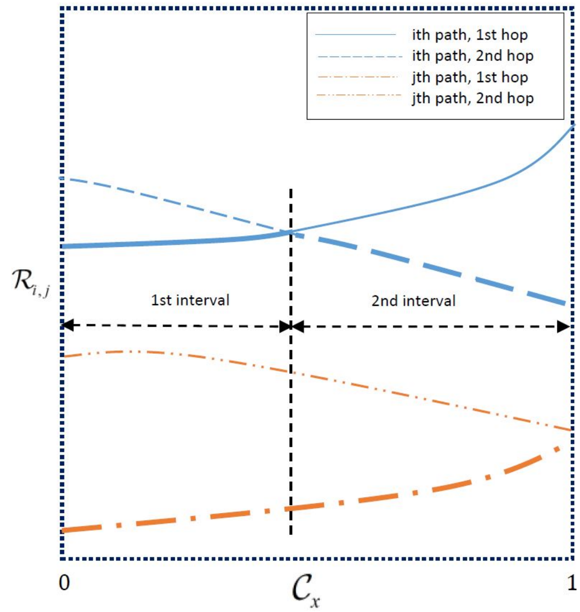


(d) The minimum rate functions are decreasing and increasing.

Fig. 1: Possibilities for the rate functions configurations in case of no intersections between the 1st and 2nd hops of both paths (solid lines for the minumum rate function).



(a) The minimum rate function of the jth path is decreasing.



(b) The minimum rate function of the jth path is increasing.

Fig. 2: Possibilities for the rate functions configurations in case of existence of intersection between the 1st and 2nd hops of only one of the paths (solid lines for the minumum rate function).

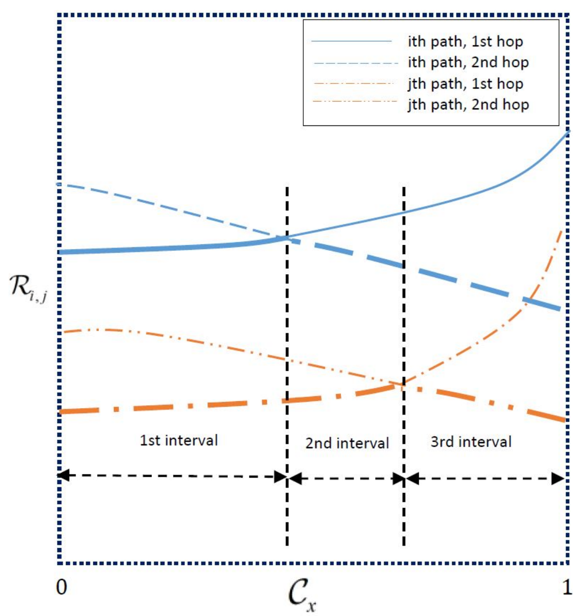


Fig. 3: Possibilities for the rate functions configurations in case of existence of intersection between the 1st and 2nd hops of both paths (solid lines for the minumum rate function).

REFERENCES

[1] M. Gaafar, O. Amin, R. F. Schaefer, and M.-S. Alouini, “Improper Signaling in Two-Path Relay Channels,” in *Proc. IEEE Int. Conf. Communications (ICC) Workshops. Submitted*, Paris, France, May. 2017.

[2] M. Gaafar, O. Amin, A. Ikhlef, A. Chaaban, and M. S. Alouini, “On alternate relaying with improper gaussian signaling,” *IEEE Commun. Lett.*, vol. 20, no. 8, pp. 1683–1686, Aug 2016.